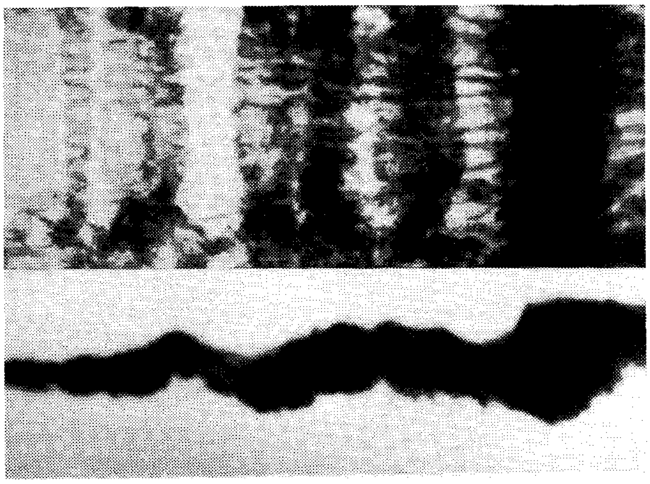

Fig. 2. Flow visualization of a reacting shear layer: simultaneous plan view (top) and side view (bottom).

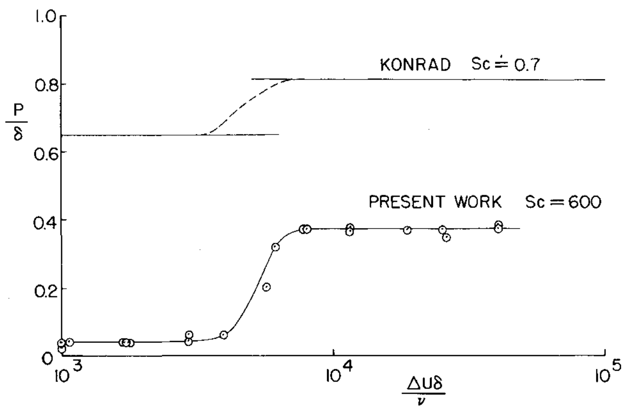


Fig. 3. Comparison of mixing in gaseous ($Sc \doteq 0.7$) and aqueous ($Sc \doteq 600$) shear layers.

The simple mixing model proposed by Broadwell[2] visualizes the mixing process as a sequence of events. The first step is the entrainment of pure, irrotational fluid into the layer. The entrained lump of fluid is subsequently broken down into smaller and smaller scales until finally the smallest scale, the Kolmogorov microscale, is reached. If diffusion is slow enough, then diffusion across the microscale will be the bottleneck in the sequence of events that culminate in molecular scale mixing. The mixing is then small scale diffusion limited.

Entrainment is visualized as the large scale engulfment of irrotational fluid by the large vortical structures.[3] These structures are assumed to behave in an inviscid manner, independent of Reynolds number; thus entrainment is also independent of Reynolds number. In the case of rapid diffusion, the model predicts that mixing occurs very soon after entrainment. The mixing is entrainment limited and thus independent of Reynolds number.

The time to diffuse across the microscale $\lambda_0$ is $\tau_{\lambda 0} \sim \lambda_0^2/D$; entrainment time is taken to be $\tau_\delta \sim \delta/\Delta U$; and the ratio of these two time scales is $T \sim Sc/Re^{1/2}$.

This model predicts that when the time parameter $T \gg 1$, the mixing is limited by small scale diffusion, whereas for $T \ll 1$ large scale entrainment is the slow process. At some intermediate value of $T$, a transition region must exist.

## Conclusion

The present results are qualitatively consistent with Broadwell's simple model. For large $Re(T \ll 1)$, the mixing is entrainment limited and independent of $Re$, while for $T \gg 1$, the mixing is limited by small scale diffusion and is thus extremely sensitive to the presence of small scales. A spanwise-sinuous wiggle is believed to be important in the introduction of the small scales. Mixing is at most only a weak function of Schmidt number at high Reynolds number. A more complete report of this and other, ongoing work is planned for the near future.

## Acknowledgment

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# Simple Recompression Model for the Korst Base Pressure Theory

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## Nomenclature

$\ell$ = coordinate along inviscid boundary
$n$ = inverse of power law exponent
$p$ = pressure
$Re_\theta$ = momentum thickness Reynolds number
$u$ = velocity
$Y$ = transverse coordinate in system following inviscid boundary
$y$ = transverse coordinate in transformed system
$y_m$ = increment of coordinate shift
$\beta$ = angle of inviscid boundary with slip line (wall)
$\gamma$ = isentropic exponent
$\delta$ = initial boundary-layer thickness
$\zeta$ = $Y/\delta$
$\eta$ = $\sigma y/\ell$ (similarity variable)
$\eta_p$ = $\sigma\delta/\ell$ (position parameter)
$\theta$ = boundary-layer momentum thickness
$\rho$ = density
$\sigma$ = similarity parameter
$\phi$ = velocity ratio $u/u_U$

*Subscripts*

$b$ = base region
$D$ = dividing streamline
$i$ = beginning of mixing
$L$ = inner edge of mixing zone
$m$ = inviscid boundary
$r$ = region downstream of shock
$S$ = stagnating streamline

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$T$ = total condition
$U$ = outer edge of mixing zone
$1$ = beginning of recompression on slip line (wall)
$2$ = end of recompression on slip line (wall)
$\infty$ = freestream

## Introduction

WHEN applying the Korst base-pressure theory, it is necessary to employ an empirical compression factor to obtain reasonable agreement with experimental data. No general method of correlation is available; however, Bauer and Fox[1] presented a simple analytical model of the recompression process which yielded a theoretical estimate of the recompression factor. This Note presents an improvement to this model. Results are shown for supersonic flow over a backward facing step.

## Base Pressure Theory

The Korst base-pressure theory is well documented in the literature[2,3]; therefore, only a brief outline of the theory and the relations of interest are presented.

The base-pressure theory is formulated by coupling three distinct analyses: 1) an inviscid analysis to determine the overall flow structure and pressure field; 2) a global analysis of the assumed flow behavior leading to the primary system of conservation equations; and 3) a viscous mixing and recompression analysis to determine key streamlines in the mixing region.

In the problem under consideration, the method of characteristics was used for the inviscid field (in anticipation of more complex flows). As the flow is considered isoenergetic, the conservation equations reduce to the mass balance equation so that from Fig. 1:

$$\int_{y_S}^{y_D} \rho u \mathrm{d}y \,|_{\text{stream 1}} + \int_{y_S}^{y_D} \rho u \mathrm{d}y \,|_{\text{stream 2}} = 0 \qquad (1)$$

The velocity profile of the mixing layer is[2]:

$$\phi(\eta) = \frac{1}{2}\left[1 + \mathrm{erf}(\eta - \eta_p)\right] + \frac{\eta_p}{\sqrt{\pi}} \int_0^1 \zeta_i^{1/n} e^{-(\eta - \eta_p \zeta)^2} \mathrm{d}\zeta \qquad (2)$$

where $\zeta_i^{1/n}$ is the initial boundary-layer velocity profile.

As shown in Ref. 2, the mixing profile must be shifted with respect to the inviscid boundary to agree better with experimental profiles so that:

$$y = Y + y_m(\ell) \qquad (3)$$

where $y$ is the coordinate used in Eq. (2), $Y$ is measured from the reference coordinate system which follows the inviscid boundary, and $y_m$ is the increment of shift, which from Ref. 2 is:

$$\eta_m = \eta_U - \eta_p + \eta_p \int_0^1 \frac{\rho}{\rho_U} \zeta^{2/n} \mathrm{d}\zeta \,|_i - \int_{\eta_L}^{\eta_U} \frac{\rho}{\rho_U} \phi^2 \mathrm{d}\eta \,|_{\eta_p} \qquad (4)$$

The dividing streamline $y_D$, which separates the main mass flux from that which is entrained from the base flow region, is taken as that which satisfies:

$$\eta_p \int_0^1 \left( \frac{\rho}{\rho_U} \zeta^{1/n} \right) \mathrm{d}\zeta \,|_i + \eta_U - \eta_p - \eta_m = \int_{\eta_D}^{\eta_U} \frac{\rho}{\rho_U} \phi \mathrm{d}\eta \,|_{\eta_p} \qquad (5)$$

There remains, then, the determination of the stagnating streamline $y_S$, which separates that part of the flow which is turned back toward the base from that which continues on through the downstream recompression.

## Recompression Analysis

With reference to Fig. 2, part of the mixing layer is assumed to turn back through an area equal to the area of approach between $y_S$ and $y_L$. The pressure rise is assumed to have the functional form given in Ref. 1 as:

$$\frac{p - p_b}{p_r - p_b} = \sin^2\left( \frac{\pi}{2} \frac{x - x_1}{x_2 - x_1} \right) \qquad (6)$$

where $x_1$ is related to $y_S$ by geometry and $p_r$ is the pressure obtained from the calculation of shock and slip line. In the present work, a momentum balance determines $x_2$.

It is assumed that the normal force due to the pressure rise on the wall is equal to the transverse momentum given up by the oncoming viscous layer. This leads to the relation

$$\int_{x_1}^{x_2} p \mathrm{d}x = \sin\beta \int_{y_L}^{y_U + y_m} \rho u^2 \mathrm{d}y + p_b (x_U - x_1) \qquad (7)$$

which is solved for $x_2$. The point $x_U$ is determined from:

$$x_U = x_m + (y_U - y_m)/\sin\beta \qquad (8)$$

Peters and Phares[4] confirm the widely held assumption that the stagnating streamline comes to rest isentropically. The streamline location $y_S$ is then determined as that location in the mixing layer where the stagnation pressure is equal to the static pressure at the downstream wall. Thus, for an isoenergetic perfect gas,

$$\phi_S^2 = \frac{1 - (p_b/p_S)^{(\gamma-1)/\gamma}}{1 - (p_b/p_{T_U})^{(\gamma-1)/\gamma}} \qquad (9)$$

## Experimental Verification

Base pressures were calculated for flows over a planar backward facing step at four freestream Mach numbers: 1.5, 2.0, 2.4, and 3.0. The similarity parameter $\sigma$ was that of Bauer,[5] all mixing calculations were made at $\ell$ where $x = x_m$ on the wall and $\eta_L$ and $\eta_U$ were taken where the velocity ratio, Eq. (2), was 0.00001105 and 0.99998895, respectively. Results

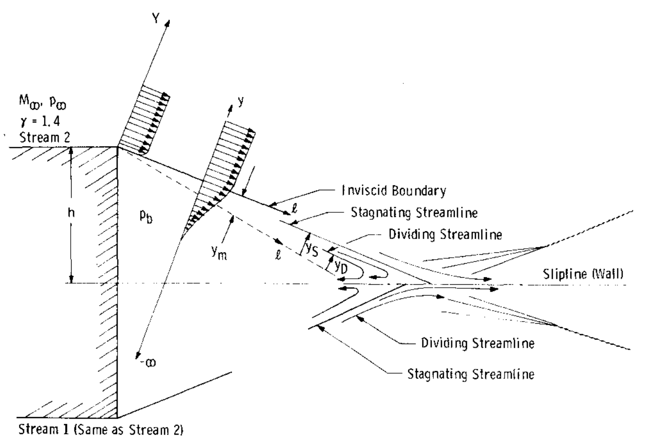


Fig. 1 Computational schematic of near wake.

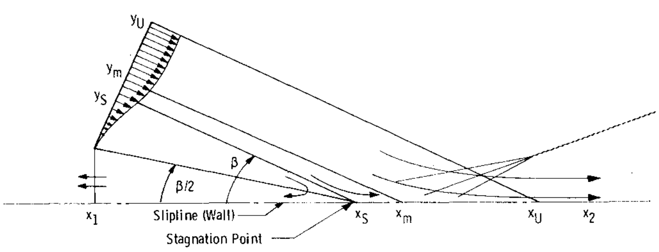


Fig. 2 Schematic of recompression region.

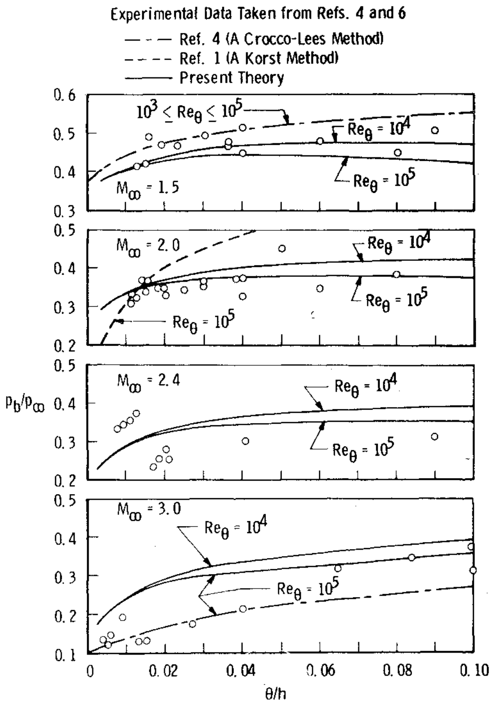


Fig. 3 Base pressure vs initial momentum thickness.

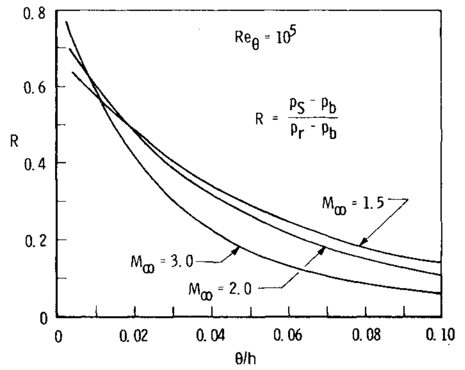


Fig. 4 Recompression factor vs momentum thickness.

of the calculations are compared with experiment in Fig. 3. The power-law exponent was taken from Ref. 4 where a correlation between $Re_\theta$ and $1/n$ is presented. The data are taken from that gathered by McDonald[6] and from Ref. 4.

Nash[7] defines a recompression factor as:

$$R = (p_S - p_b) / (p_r - p_b) \tag{10}$$

This factor, corresponding to the theoretical results of Fig. 3, is presented in Fig. 4.

## Acknowledgment

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# Singularity at the Trailing Edge of a Swept Wing

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## Introduction

In calculations of three-dimensional transonic flows, the most commonly applied method has been that of transonic small-perturbation theory, solving the resulting equations by finite differences. Examples of the solution of the full equations of motion applying the boundary conditions exactly, have been few, e.g., Duck,[1] Jameson,[2] Clapworthy,[3] Forsey and Carr.[4] The results of some of these are applicable only in certain specialized cases.

The general philosophy of these methods is to introduce a coordinate system defined by the shape of the wing, so that the boundary conditions may be satisfied exactly (in the numerical sense). One manner of doing this, which has been used successfully in Refs. 3 and 4, is to define the set of vertical planes that are parallel to the flow, to be coordinate surfaces; the transformation to body-defined coordinates is then completed by performing the conformal mappings of the wing section on to a unit circle in each of these planes. This coordinate system will, in general, be nonorthogonal.

The existence of numerical conformal mapping techniques means that the mappings will usually be performed numerically. However, when the method was used for swept wings with a Kármán-Trefftz profile, the mappings for which can be expressed analytically, it was discovered that there are certain difficulties associated with the trailing edge. The use of numerical conformal mappings would, almost certainly, result in these difficulties being overlooked, and it is the purpose of this Note to draw attention to their existence.

## Coordinate System

We consider a symmetrical wing with a Kármán-Trefftz profile, swept at an angle $\Lambda$, at zero incidence to the freestream. The Cartesian coordinate system $x^i$ has $x^1$ spanwise, $x^2$ upstream, and $x^3$ vertically upward. From this, we transform to a nonorthogonal reference system $\xi^i$ in which the center section is represented by $\xi^1 = 0$, planes parallel to it by $\xi^1 =$ constant, and the wing surface by $\xi^2 = 0$.

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